EGC 220 **Practice Problems for Exam 1**

1. Circle T (true) or F (false) for each of these Boolean equations.
Solution:

(a).	T	F	$A + 1 = A$
(b).	T	F	$A + BC = (A + B)(B + C)$
(c).	T	F	$\overline{A} \oplus \overline{B} = A \oplus B$
(d).	T	F	$A(BC) = (AB)C$
(e).	T	F	$\overline{A + B + C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

2. Evaluate the following:

2(a). Convert to hex: $11001001.1011_2 = ?$

Solution:

Integer Part: $1100_2 = CH$ $1001_2 = 9H$
Fractional Part: $1011_2 = BH$
Verification of Fractional Part: $1011_2 = 1 \times 2^{-1} + 1 \times 2^{-3} + 1 \times 2^{-4} = 1/2 + 1/8 + 1/16 = 8/16 + 2/16 + 1/16 = 11/16 = 11 \times 16^{-1} = B \times 16^{-1}$
Verification is for your own interest; you are not expected to show this on an exam problem.

$\boxed{11001001.1011_2 = \text{C9.BH}}$

2(b). Convert to decimal: $110101.01_2 = ?$

Solution:

Integer: $110101_2 = 1 \times 2^0 + 1 \times 2^2 + 1 \times 2^4 + 1 \times 2^5 = 1 + 4 + 16 + 32 = 53_{10}$
Fraction: $.01_2 = 1 \times 2^{-2} = 1/4 = 0.25_{10}$

$\boxed{110101.01_2 = 53.25_{10}}$

2(c). Convert to binary: $98.25_{10} = ?$

Solution:

Using sum-of-powers method, we obtain for the integer and fractional parts:
Integer:

- $2^6 = 64_{10} < 98_{10}$
- $98_{10} - 64_{10} = 34_{10}$
- $2^5 = 32_{10} < 34_{10}$
- $34_{10} - 32_{10} = 2_{10}$
- $2^1 = 2_{10}$
- $2_{10} - 2_{10} = 0_{10}$

Integer: $98_{10} = 2^6 + 2^5 + 2^1 = 1100010_2$
Fractional Part:

- $2^{-2} = 1/4 = 0.25_{10}$
- $0.25_{10} - 0.25_{10} = 0$

Fraction: $0.25_{10} = 2^{-2} = 0.01_2$
Note: It is sufficient to write that $0.25_{10} = 2^{-2} = 0.01_2$ for the fractional part.

$\boxed{98.25_{10} = 1100010.01_2}$

2(d). Convert to decimal: $2A.4_{16} = ?$

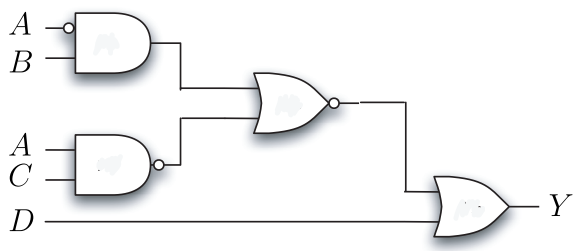
Solution:

Either convert directly from hex:
$2A.4_{16} = 2 \times 16^1 + A \times 16^0 + 4 \times 16^{-1} = 32 + A + 4/16 = 32 + 10 + 1/4 = 42.25_{10}$

or convert to binary first and then to decimal:
$2A.4_{16} = 0010\ 1010.0100_2 = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^{-2} = 32 + 8 + 2 + 1/4 = 42.25_{10}$

$\boxed{2A.4_{16} = 42.25_{10}}$

3. Write the Boolean expression equivalent to the following logic circuit. Do not simplify!



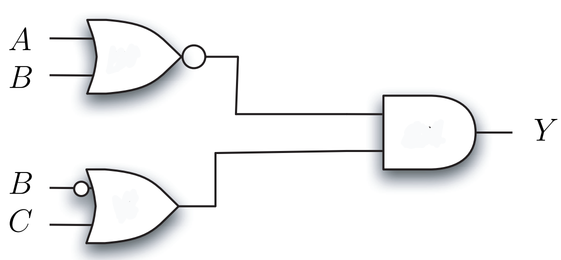
Solution:

$$\boxed{Y = \overline{(\overline{A} \cdot B + \overline{A \cdot C})} + D}$$

4(a). Draw the logic circuit realization of the following Boolean expression as stated. Do not simplify! You may draw inverters explicitly or use inversion bubbles, as you choose.

$$Y = f(A, B, C) = \overline{(A + B)}(\overline{B} + C)$$

Solution:



4(b). Write the complete truth table for the Boolean expression of 4(a).

Solution:

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

4(c). Convert the Boolean equation of 4(a) to its DeMorgan equivalent.

Solution:

There are two (very similar) ways to apply DeMorgan's theorem to 4(a).

$$
\begin{aligned}
\overline{(A+B)}(\overline{B}+C) &= \overline{\overline{\overline{(A+B)}(\overline{B}+C)}} \\
&= \overline{\overline{\overline{(A+B)}} + \overline{(\overline{B}+C)}} \\
&= \overline{(A+B) + \overline{(\overline{B}+C)}} \\
&= \overline{(A+B) + \overline{\overline{B}}\cdot\overline{C}} \\
&= \overline{(A+B) + B\cdot\overline{C}}
\end{aligned}
$$

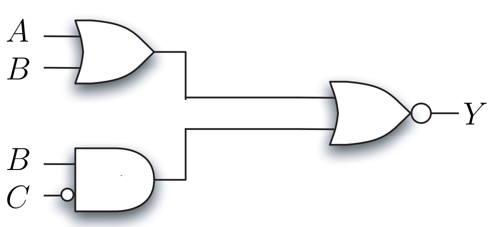
OR

$$
\begin{aligned}
\overline{(A+B)}(\overline{B}+C) &= \overline{\overline{\overline{(A+B)}(\overline{B}+C)}} \\
&= \overline{\overline{\overline{(A+B)}} + \overline{(\overline{B}+C)}} \\
&= \overline{\overline{(\overline{A}\cdot\overline{B})} + \overline{\overline{B}}\cdot\overline{C}} \\
&= \overline{\overline{(\overline{A}\cdot\overline{B})} + B\cdot\overline{C}} \quad \leftarrow \text{ stopping here gives circuit 4.d.(alt)} \\
&= \overline{(\overline{\overline{A}}+\overline{\overline{B}}) + B\cdot\overline{C}} \\
&= \overline{(A+B) + B\cdot\overline{C}}
\end{aligned}
$$

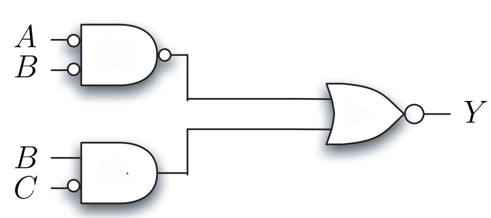
$\boxed{Y = \overline{(A+B)+B\cdot\overline{C}}}$ OR $\boxed{Y = \overline{\overline{(\overline{A}\cdot\overline{B})}+B\cdot\overline{C}}}$ for SOP

4(d). Draw the logic circuit for the DeMorgan equivalent Boolean equation you found in 4(c). You may use inverters or inversion bubbles, as you choose.

Solution:



OR



(alt)

5. Simplify the following Boolean expression as far as possible, using the postulates and theorems of Boolean algebra. _DO NOT_ use a Karnaugh map except possibly to check your work. You do not have to justify each step by stating the theorem or postulate used, but you must show each step in your simplification.

$$f(w,x,y) = w\overline{x}y + wx + w\overline{y} + wx\overline{y}$$

Solution:

$$\begin{array}{lll}
w\overline{x}y + wx + w\overline{y} + wx\overline{y} & = w\overline{x}y + wx + w\overline{y} + w\overline{y}x & \text{associative (b)} \\
 & = w\overline{x}y + wx + w\overline{y}\cdot 1 + w\overline{y}x & \text{identity (b), } A\cdot 1 = A \\
 & = w\overline{x}y + wx + w\overline{y}(1+x) & \text{distributive (b), } A(B+C) = AB + AC \\
 & = w\overline{x}y + wx + w\overline{y}\cdot 1 & \text{Nullity Thm (a), } 1 + A = 1 \\
 & = w\overline{x}y + wx + w\overline{y} & \text{identity (b)} \\
 & = w(\overline{x}y + x) + w\overline{y} & \text{distributive (b)} \\
 & = w(x+y) + w\overline{y} & \text{Reduction Thm (a), } A + \overline{A}B = A + B \\
 & = w(x+y+\overline{y}) & \text{distributive (b)} \\
 & = w(x+1) & \text{complement (a), } A + \overline{A} = 1 \\
 & = w\cdot 1 & \text{Nullity Thm (a)} \\
 & = w & \text{identity (b)} \\
w\overline{x}y + wx + w\overline{y} + wx\overline{y} & = w & \text{Simplified!}
\end{array}$$

$\boxed{f(w,x,y,z) = w}$

Note: You may approach this problem in a different order, for example, by factoring out w initially. This is fine, as long as you include all steps and obtain the same final answer.

6. Simplify the following expression using the postulates and theorems of Boolean algebra. Eliminate all group complements. Justify each step by stating or referrring to the Boolean theorem or postulate you use. Don't skip any steps! Do NOT use a Karnaugh map to simplify the expressions.

$$\overline{(A\cdot B\cdot C)}(A+C)(A+\overline{C})$$

Hint: Remember DeMorgan's theorem!

Solution:

$$\begin{array}{lll}
\overline{(A\cdot B\cdot C)}(A+C)(A+\overline{C}) & = \overline{(A\cdot B\cdot C)}\cdot A & \text{Combining Thm (b), } (A+B)(A+\overline{B}) = A \\
 & = (\overline{A} + \overline{B} + \overline{C})\cdot A & \text{DeMorgan's Theorem, } \overline{A\cdot B} = \overline{A} + \overline{B} \\
 & = A\overline{A} + A\overline{B} + A\overline{C} & \text{distributive (b)} \\
 & = 0 + A\overline{B} + A\overline{C} & \text{complement (b), } A\cdot\overline{A} = 0 \\
 & = A\overline{B} + A\overline{C} & \text{Simplified!}
\end{array}$$

$$\overline{(A\cdot B\cdot C)}(A+C)(A+\overline{C}) \quad = A\cdot\overline{B} + \mathrm{A}\overline{C})$$

7. Given $Y = f(w,x,y,z) = \prod M(0,1,3,5,13)$,

7(a). Write the complete truth table for $Y = f(w,x,y,z)$.

Solution:

w	x	y	z	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

7(b). Write $Y = f(w,x,y,z)$ in POS canonical form. (Do not use Σ or Π notation in your final answer.)

Solution:

$$\boxed{Y = (w+x+y+z)(w+x+y+\overline{z})(w+x+\overline{y}+\overline{z})(w+\overline{x}+y+\overline{z})(\overline{w}+\overline{x}+y+\overline{z})}$$

7(c). Write $Y = f(w,x,y,z)$ in **shorthand** SOP form. (Use Σ or Π notation in your final answer.)

Solution:

$$\boxed{Y = \sum m(2,4,6,7,8,9,10,11,12,14,15)}$$

7(d). Write $Y = f(w,x,y,z)$ in SOP canonical form. (Do not use Σ or Π notation in your final answer.)

Solution:

$$\begin{aligned} Y =& \overline{w}\,\overline{x}\,y\overline{z} + \overline{w}\,x\,\overline{y}\,\overline{z} + \overline{w}\,x\,y\,\overline{z} + \overline{w}\,x\,y\,z + w\,\overline{x}\,\overline{y}\,\overline{z} + w\,\overline{x}\,\overline{y}\,z + w\,\overline{x}\,y\,\overline{z} \\ &+ w\,\overline{x}\,y\,z + w\,x\,\overline{y}\,\overline{z} + w\,x\,y\,\overline{z} + w\,x\,y\,z \end{aligned}$$